

The Spatial Patterns of Supply and Demand in Health Services

The Case of the Médio Tejo Region

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Abstract

In rural areas, the supply of health services depends not only on the quality of human and capital investments, but also on the spatial distribution of infrastructures. This will greatly determine patients' accessibility to health care. In this paper, taking into account the hierarchical structure of the National Health Service, the spatial distribution of health infrastructures is compared with the patients' spatial distribution in order to measure market efficiency for the sector in one particular rural region. In terms of methodology, a raster GIS is built in order to allow the researcher to use different measures of the spatial surface under analysis.

Keywords: Health services, Accessibility, Zipf Law, Raster models.

1 Introduction

Access to health care and the quality of the services provided is of the topmost importance when evaluating the efficiency of the National Health System. However, access in this area of study is not simply a function of the spatial distribution of health infrastructures, it depends on economic, social-demographic, psychological and cultural variables as well as demographic (Santana, 1995).

The main objective of the present study is to explore different methodologies for calculating the geographical portion of the accessibility equation; hence, aware of the complexity that this issue carries, for the remain of the paper, when the word accessibility is mentioned, it will refer to time distances to particular health system infrastructures.

The idea that demand meets supply for health services (Santana, 1995) will be challenged; there is little evidence that given an uneven spatial structure, distribution of health infrastructures can meet Says Law; even in the long run, it is a difficult argument to accept. In order to further explore this argument, the system's capacity for each of the three district hospitals will be calculated; this will serve as a measure of supply.

For district hospital i , the area of influence, or market area, is defined as the proportion of the spatial surface for which the nearest hospital is i . Market areas for the 3 district hospitals of the Médio Tejo region are calculated based on the accessibility algorithm explained below.

The rest of the paper will develop as follows: section 2 will present the target region, as well as discuss some important theoretical issues concerning the measurement problem; section 3 starts with the discussion and presentation of the different methods used for calculating accessibility surfaces; after, the issue of model selection will be discussed; the section ends with the issue of supply versus demand, where the estimated data will serve as input for calculating supply capacity; section 4 concludes.

2 Background Information

The target area is the Médio Tejo region, located in the north corner of the old Lisbon and Tagus Valley NUTS2 region. It has 10 local councils and a population of around 225000. It is important to note that human settlements occupy only around 19% of the spatial surface; it is mostly a rural region made up of a few middle-size towns, and a large number of small villages, where agriculture plays an important role in the local economies.

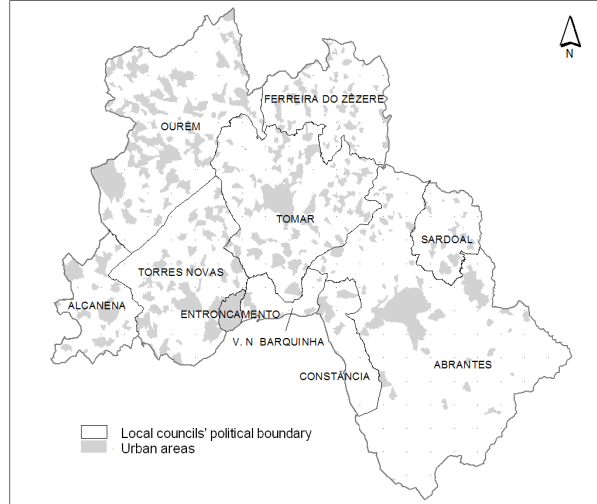


Figure 1: medioTejo

The Médio Tejo region has three district hospitals, 16 health centres and 48 health centre extensions. One characteristic of the Portuguese health system is the little vertical articulation between layers; hence, access to all three structures is important.

In the present study, supply will be measured as the capacity of the health system weighted by a distance function; this will be similar to the traditional specification of regional potential (Harris, 1954); this implies the calculation of some quotient, where in the numerator some measure of dimension or capacity is used while some function of distance is the most common denominator. In its general form, regional potential can be represented as:

$$s_i = g(\cdot) \sum_{j=1}^N d_{ij}^{-\alpha}, \quad (1)$$

where s_i represents potential of unit i , $g(\cdot)$ is a function of dimension or development and $d_{ij}^{-\alpha}$ represents the distance between any origin in the spatial surface and unit i .

The specification of $g(\cdot)$ varies according to the objective of different studies. In the present paper, a measurement of dimension and to some extent quality of health services supplied will be used. Quality of health services in this case will be taken as simply a function of hospital capacity, given that the introduction of more complex measures would go far beyond the scope of the paper and would simply introduce extra sources of non-stochastic noise in the model.

In relation to the distance between any origin and a given infrastructure, it is assumed again for simplicity purposes that individuals use mainly private means of transport to access the nearest health infrastructure. This assumption is not distant from reality because public transport is scarce in rural Portugal.

Distances are calculated taking into consideration the existing road network and speed limits according to road types. A raster GIS is built in order to take into consideration origins away from the network. Different specifications are calculated and later tested. This is needed in order to parameterize the model in the most rigorous way possible.

The α parameter introduced in (1) represents the impedance exerted by distance in relation to the flow of individuals. It would be possible to use some theoretical coefficient, a methodology common in gravity studies. In this case, the α coefficient will be estimated beforehand using for this purpose the gradient of the regional urban hierarchies. For this purpose, a specification similar to the *rank – size* rule will be estimated. As it is well known, this is an adaptation of the Zipf Law (Gabaix et al. 2003), used to represent the empirical relation between urban size and their ranking position in a particular regional structure. The specification used takes the following form:

$$P_n = P_1 (n)^{-\alpha}, \quad (2)$$

where P_n represents population of unit n , part of the urban hierarchy made up of units $N = \{1, 2, \dots, N\}$. Urban centres are ordered according to population; P_1 represents the number of individuals living in the largest spatial unit, P_2 the same for the second largest, etc.

As said before the *alpha* coefficient will be used in order to calibrate the measurement of each health infrastructure capacity. This is central because the value calculated in (1) will be used as a measure of supply for the following analysis.

3 Model Specification

In order to calculate the distance between each National Health Service infrastructure and potential demand sites, it is necessary to build accessibility surfaces using a raster GIS. For this purpose cost surfaces were built based on the speed limits of each vector belonging to the road network. Speed limits were transformed into traveling cost per meter. The traveling cost for cells outside the road network were calculated using a 6 Km/h speed (Godinho Rodrigues 2000a & 2000b, Julião 1998). Using this methodology, cell crossing times were calculated using the expression:

$$tc_{mn} = \frac{c \times 60}{vel \times 1000}, \quad (3)$$

where tc_{mn} represents the time it takes to cross cell mn and c represents cell size. The criteria for choosing the cell size of the cost surface must always be a balance between the availability of computing power and the attempt to be as rigorous as possible. In this study,

the structure of each surface is a 25*25 meter grid. The following table represents the different type of road, te associated speed limit and cell crossing time.

Road Type	Speed Limit	Cell Crossing Time
Motorway	120	0,0005
Main National Roads	90	0.000667
National Roads	70	0.000857
National Roads Crossing Urban Areas	50	0.0012
Cell outside road network	6	0.01

Based on the cost surfaces built using these values, the aggregate minimum traveling cost from each cell of each cost surface to the nearest destination (ex. nearest hospital) was calculated. The resulting accessibility surfaces were then used to find average traveling cost to each infrastructure and from each administrative area.

3.1 Cost Surface Applications

In this study, there was a need to calculated accessibility to three levels of health service infrastructures: district hospitals, health centres and health centre plus extensions. District hospitals constitute the secondary health system infrastructures whilst health centres and their extensions constitute the primary health system infrastructures.

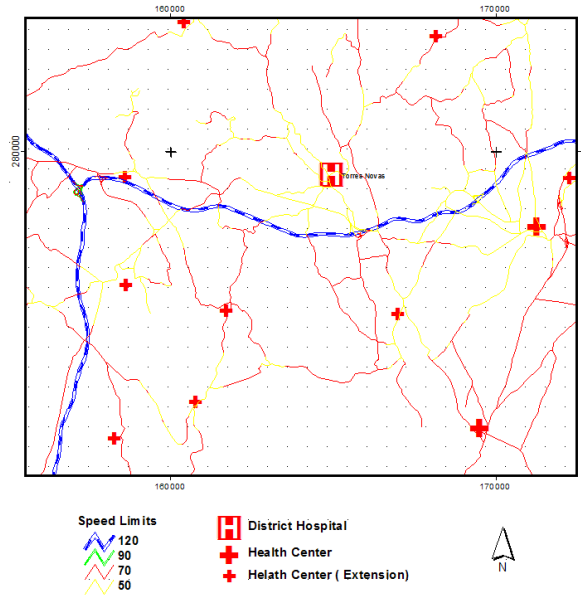


Figure 2: Health Infrastructures

For each of the three level, three different methods were used: the first two methods use all the spatial surface as potential sources of demand for health services. The difference between the two is that the first model uses a more crude calculation where it is assumed that it is possible to enter any road at any point; in the second model, agents/individuals are forced

to enter the motorway only at an existing node¹. In the third model, sources of demand are restricted to urban areas selected from census tracts. This reduces greatly the size of the cost surface, while approximates the model to the existing settling patterns.

Model 1: The *purist* way of calculating accessibility

As said before, accessibility was calculated using three sets of possible destinations. Figure 3.1 represents a sample of the spatial surface with the distribution of the different levels of health infrastructures. The implementation of the model was done using only one run of the algorithm, as no constraints on network entry were imposed². Table 3.1 presents the average traveling time from any location within each local council³. Naturally, as there are more health centers and even more extension, distances to the nearest infrastructure decrease once we consider primary system infrastructures.

Local Council	Mean Accessibility (Min.)		
	Hospitals	Health centers	Health centers + extensions
Abrantes	27.6	23.7	15.4
Alcanena	19.4	12.2	7.3
Constância	25.9	20.7	12.5
Entroncamento	10.0	5.2	5.1
Ferreira do Zêzere	27.0	14.7	10.3
Sardoal	20.6	12.7	10.4
Tomar	14.7	14.3	8.6
Torres Novas	14.0	13.4	8.8
V. N. Barquinha	15.8	8.7	7.7
Ourém	27.1	21.3	10.0

Model 2: Introducing Network Constraints

Although it is assumed that the network used is not complete, as there are numerous regional roads linking small neighborhoods and small rural settlements, it is thought that the model would further approximate reality if individuals traveling on the network were constrained to enter any motorway only in any existing motorway node. This adds a new run of the algorithm when implementing the model. This two steps approach works as follows: first, accessibility from motorway nodes to health infrastructures are calculated; second, accessibility from any location of the cost surface to the nearest node or health infrastructure is computed. Finally, to this later surface, time distances from the nearest node⁴ are added in order to obtain total travel distance in minutes as before.

¹Note that more complex models could be used, forcing each agent not to enter the road network if nor at a crossing; however, given that the main advantage of using a raster model is to consider all areas where individuals live, further constraints would render the model too complex, too abstract and with little potential advantages over standard vector models.

²Details of the cost and accessibility surfaces are presented in the end of the document.

³Seconds are presented using a decimal scale.

⁴zero in the case of health infrastructures

Local Council	Mean Accessibility (Min.)		
	Hospitals	Health centers	Health centers + extensions
Abrantes	29.0	22.9	16.1
Alcanena	19.6	11.7	8.1
Constância	29.3	19.8	13.1
Entroncamento	10.8	4.8	5.3
Ferreira do Zêzere	28.1	14.2	11.2
Sardoal	21.5	12.5	11.4
Tomar	15.9	12.1	8.8
Torres Novas	15.0	12.7	9.5
V. N. Barquinha	16.5	7.2	7.2
Ourém	30.6	21.6	11.0

Comparing tables 3.1 and 3.1 it can be observed that mean accessibility values increase using this second methodology; this results from the imposition of the referred constraint on movement.

Model 3: Introducing Surface Constraints

Using raster data format to calculate accessibility carries the advantage of making endogenous to the model locations outside the road network. This however, taken to an extreme, can be seen as a disadvantage as areas with no human settlements are included in the cost surface and in this case considered as potential sources for demand for health services. In previous studies (GODINHO RODRIGUES 1999, 2001), this problem was taken into account by buffering the network and considering only location inside the buffers; furthermore, accessibility values for cells outside the road network were interpolated in such a way as to cause the time distance to increase as we move away from any road. This greatly enhanced the model, but was done mainly because geographical data on human settlements at a large scale were not available.

In the present study, urban areas were chosen with great detail using the census 2001 tracts, and carefully checking these data against raster topographical maps at a 1:25000 scale. Details of the urban areas are shown in figure 3.1.

3.2 Model Selection in Accessibility Analysis

The methods for calculating distances to health service infrastructures produce different results according to the constraints introduced or not. These differences, although apparently small, when used to calculate 2nd order coefficients may give rise to distortions if one does not choose the most accurate specification.

Instinct would tell that by introducing network and surface constraints, the researcher approximates reality; on the other hand, restrictions may condition the model, and one could argue that the first model, which takes a more parsimonious approach to cost surface modeling captures the freedom of movement individuals have, and that more complex specifications do not add any valuable information.

As a way of testing the three models, the accessibility gradient for each model, concerning traveling costs to district hospitals, will be tested against the gradient of a Zipf Law specification, estimated for the NUTS5 spatial units of the region. The choice of this regional disaggregation level and not a finer scale, using settlements data is that the latter do not have any administrative meaning. The other alternative would have been to use the local councils data, but at the scale of the present study, these are thought not to capture differences between smaller units, hence the problem of *ecological fallacy* would have been greater.

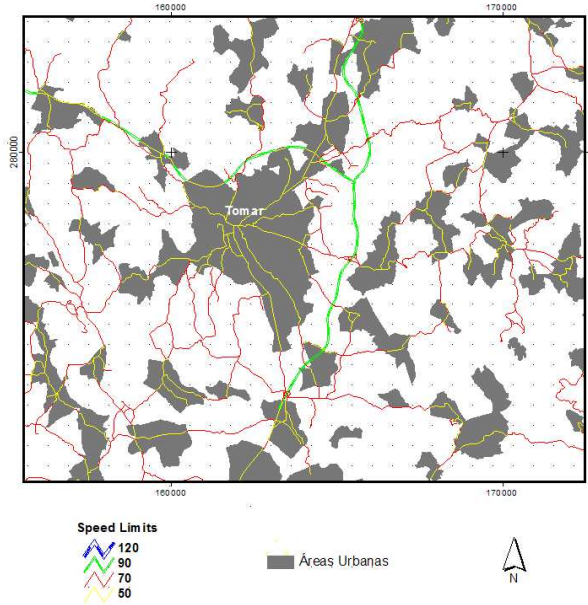


Figure 3: Urban areas

The Zipf specification, when tested against urban population data is normally known as the Rank-Size specification. Equation (2) was transformed by taking the logs on both sides. The resulting specification is given by:

$$\log(n) = Const + \alpha \log(P_n) , \quad (4)$$

where $\log(P_n)$ is the logarithm of population of urban centre of order n , $\log(n)$ is the logarithm of the order of the urban center, the constant term should approximate the logarithm of the population of the leading region; α is known as the Zipf coefficient, and its module should approach unity. The model was estimated using Ordinary Least Squares (OLS)⁵. Table 3.2 presents the values of the Zipf coefficient for the three hierarchies⁶.

	Zipf Coef.	Sig.	R-square
Tomar	-1.044793	<0.0000	0.9556
Abrantes	-0.998818	<0.0000	0.9114
Torres Novas	-0.961905	<0.0000	0.9433

The estimation of the Rank-Size specification is not free of problems⁷; the most important of these concern the bias in the Zipf coefficient. These issues will not be dealt in the present

⁵When building the accessibility surfaces for the hospitals destinations, sub-regions are also defined as the result of the algorithm search patterns; these new spatial units represent the locations within each district hospital market area. The basic assumption is that individuals in need will search the nearest infrastructure taking the existing road network into account. NUTS5 regions are assigned to a specific hospital according to these sub-regions. Population for each unit for each sub-region is used in different estimations.

⁶For each run of the model, since the cost-surface used is different, the assignment of NUTS5 units to each sub-region could potentially differ; this is not the case for the first two models, since no change in the spatial sources of demand is done. In relation to model 3, only existing urban settlements are considered, which in fact change the number of NUTS 5 regions assigned to each hospital. However, differences in the Zipf coefficients are meaningless

⁷For an extensive review of estimation techniques of the Rank-Size Rule see Gabaix...

paper for two reasons: first, the objective of the exercise is to compare gradients estimated using the same technique; hence, any bias in the slope coefficient will be translated in the same form in all estimations. The second reason is that there is not any agreement about which alternative to OLS or indeed to the specification presented should be used. Therefore, the most parsimonious approach in this case suffices.

For the different accessibility surfaces, as said before, time distances from district hospitals were aggregated and by NUTS5 regions and the slope of the Zipf specification estimated using OLS.

	Zipf Coefficients		
	Model 1	Model 2	Model 3
Tomar	-1.5965	-1.7068	-1.4109
Abrantes	-1.5201	-1.4702	-1.5667
Torres Novas	-2.0301	-2.1457	-1.9103

The results presented in table 3.2 show that the regularity suggested by the Zipf Law is lost in what distribution of health services are concerned⁸. It is not the purpose of the present paper to explore this matter further, but it is nonetheless important to note that the fact that all Zipf coefficients are greater than unity in absolute terms means that there is a strong bias towards urban areas. These results are easily explained by the fact that district hospitals are located in the main urban nodes.

What matters most in the present case is to compare the results obtained with the slope coefficients obtained with the population estimations. Differences between the three models are sufficient to say that model 3, whose specification uses surface constraints is the one which approach best the results presented in table 3.2. Hence, time distances estimated from this equation are used for further calculations.

As mentioned in the introduction, demand and supply patterns in health services are not assumed to coincide; in fact, it is one of the goals of the present study to explore how they differ. However, this does not contradict the fact that population distribution is used as a benchmark to access the performance of the accessibility algorithm. It is nonetheless acknowledged that more robust techniques would help when comparing methods of calculating distances between urban centres. Nonetheless, The universal nature of the Zipf specification and the fact that it is a "theory free" specification justify its use in this line of research.

Supply Capacity versus Potential Demand

We turn now to the calculation of a coefficient that falls within the family of regional potential coefficients. The main goal is to estimate the level of supply of health services in the Médio Tejo region; with this purpose, supply will be calculated for each of the sub-regions under the influence of one of the three district hospitals present (market areas). Following expression 1, this will take the form of some function of the existing infrastructures capacity weighted by distances. Capacity in this case will be a function $g(\cdot)$ which will be a linear combination of the number of beds in the district hospital ($Hosp$), the number of health centres (HC) and the number of health centre extensions (HcX). This will take the form:

$$g_h(Hosp, HC, HcX) = \frac{1}{area_h} (w_1Hosp_h + w_2HC_h + w_3HcX_h) , \quad (5)$$

where $w_i = \{w_1, w_2, w_3\}$ are the weights of each of the variables. Note that in this study each variable is given an equal coefficient. Selection of weights and indeed variables which

⁸All coefficients are significant with probability less than 0.001.

determine capacity of a Health system a vital, and indeed should be done with care. Since the main goal of the present paper is to explore different methods of weighting these family of variables, other coefficients were not tested. In expression 5 all variables are weighted by the total area of the spatial surface under the influence of hospital h . Expression 1 will now look like:

$$\tau_i = g(Hosp, HC, HCX) \sum_{j=1}^N d_{ij}^{-\alpha}, \quad (6)$$

where i is the location of each hospital, and j is the location of any other location. Distances were calculated using the third method explained above since this was found to be the most robust. The value of α will be that which was found in the Zipf estimations for each of the hospitals (see table 3.2).

Two other weights were added to the measure of regional health system capacity; the first concerns the fact that as a rule health centres and in particular health centres extensions are associated to particular administrative unit. Hence, the area of each NUTS5 spatial unit ($Area_{freq}$) should approach the market area of each extension ($Area_{ext}$). This follows the fact that most of the NUTS5 units have their own health centre or health centre extension. For the market area of each district hospital, the Pearson correlation coefficient between ($Area_{freq}$) and ($Area_{ext}$) will be added to expression 8. In some sense, this measure the efficiency of the existing political boundaries.

The second weight added follows the quantitative geography tradition and attempts to quantify the effect of regularity in shape and size of spatial units and include this as a measure of efficiency. We define some function $\Gamma(\cdot)$ which depends positively on Maximum Distance needed to travel to the nearest health centre or health centre extension ($Dmax_i$) and negatively on the size of the market area of each infrastructure (A_i). The smaller the value of γ_i the greater the spatial efficiency of unit i . This variable takes the form $\gamma_i(Dmax, A) = \frac{Dmax_i}{A_i}$. The mean for each district hospital was then calculated; the final specification for measuring efficiency of the spatial structure takes the form:

$$\gamma_{i,h} = \frac{1}{N} \sum_{i=1}^N \frac{Dmax_i}{A_i} \quad (7)$$

for N health centres and health centre extensions in the market area of hospital h . Γ is assumed to be inversely related to efficiency; hence, it should be included in the denominator of the final capacity specification.

The augmented regional health system capacity measure, after adding the variables just mentioned, takes the form:

$$\tau_i = g_i \sum_{j=1}^N d_{ij}^{-\alpha} \cdot \rho_i \cdot \frac{1}{\gamma_i} \quad (8)$$

This expression, when applied to the three district hospitals' market areas of the Médio Tejo Region produces the following results:

	τ
Abrantes	5.42
Tomar	2.00
Torres Novas	2.53

The main conclusion which may be drawn from table ?? is that the Hospital located in Abrantes is the one which, together with the primary means of health service infrastructures, covers its market area in the most efficient way. Two important comments are in order: first, the main reason for this result favoring Abrantes is the large number of beds available and the area covered (larger than the other two hospitals⁹). The second comment concerns the fact that results would have been quite different if any of the other two models for calculating the accessibility surfaces were chosen; the reason for this is that a large proportion of the Abrantes hospital market area is not urbanized; this large proportion of the original cost surface for reasons explained before is not considered in model three, the one found to be more robust. For any of the other two models, time distances for the Abrantes region are far greater since all spatial surface is considered. This point enhances the important role given by the authors to methodological issues concerning the building of accessibility surfaces.

Finally, one needs to compare this indicator with the population variable for each market area; as mentioned before, the former measures supply capacity while the latter measures potential demand. If the observations for both indicators (τ and population) are standardized¹⁰, the results for the three market areas are best analyzed graphically (see figure 3.2).

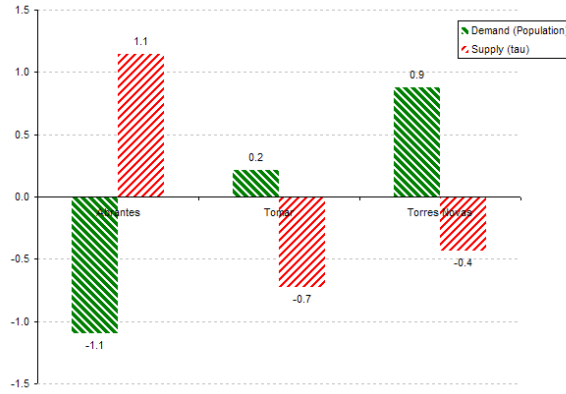


Figure 4: Supply vs. Demand (three hospital market areas)

It can easily be observed that Abrantes, which scores best in terms of supply capacity, is the sub-region with the least potential demand; this fact causes a market disequilibrium in the region. It is necessary to emphasize that the analysis performed is a rather simplistic one. It serves mainly to pinpoint possible methods to explore disequilibrium structures in the market for health services. Using the same methods, further analysis may be performed at other scales, and indeed with different weights for the capacity function g .

4 Conclusions

The objective of the paper was to present different methodologies which may be used for calculating the spatial aspect of the health services equation. In the field of Health Geography, there is a large body of research which explores the nature of demand through the demographic,

⁹In the appendices, a detailed list of all other values for the variables used will be shown.

¹⁰ $z_{\tau} = \frac{\tau_i - \bar{\tau}}{s_{\tau}}$, where $\bar{\tau}$ is the sample mean of τ and s_{τ} is the standard deviation of τ ; $z_{pop} = \frac{pop_i - \bar{pop}}{s_{pop}}$, where pop_i represents population of region i , \bar{pop} is the sample mean of pop and s_{pop} is the standard deviation of pop

social, economic and psychological characteristics of potential patients. The contribution of the present research is mainly to explore new ways of using GIS technology to aid further research studies concerning the spatial structure of the regions being examined. The authors attempted also to explore ways of testing the robustness of different methods of calculating time distances; this was done through the comparison of Zipf coefficients obtained using data aggregated from accessibility surfaces with Zipf coefficients estimated using population data (using the standard rank-size specification).

An application of the methodology with the aim of comparing supply and demand of health services in the Médio Tejo region was attempted using a specification inspired in the regional potential equations, which takes into account capacity and distances; two other weights were added to take into account the shape and size of spatial units and the distortions caused by political boundaries.

Further research is needed for different geographical datasets in order to use other capacity variables; this is acknowledged to be the greatest weakness of the dataset used. The measurement of time distances as well as the robustness tests are thought to be efficient, although alternative estimation methods for the Zipf equation should be performed. Nonetheless, this contribution aimed mainly to explore the potential use of raster models as a useful framework for exploring the spatial structures of agents interacting inside the existing system of supply and demand for health services.

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Appendix 1 - Zipf estimation for Models 1, 2, 3

Abrantes (Model1)

Ordinary Least-squares Estimates

R-squared = 0.6355

Rbar-squared = 0.6203

$\sigma^2 = 0.0503$

Durbin-Watson = 0.3728

Nobs, Nvars = 26, 2

Variable Coefficient t-statistic t-probability

Const 3.013280 9.696190 0.000000

Zipf -1.520137 -6.468500 0.000001

Tomar (Model 1)

Ordinary Least-squares Estimates

R-squared = 0.7256

Rbar-squared = 0.7173

$\sigma^2 = 0.0397$

Durbin-Watson = 0.1186

Nobs, Nvars = 35, 2

Variable Coefficient t-statistic t-probability

Const 3.193893 14.380169 0.000000

Zipf -1.596452 -9.340835 0.000000

Torres Novas (Model 1)

Ordinary Least-squares Estimates

R-squared = 0.7920

Rbar-squared = 0.7855

$\sigma^2 = 0.0300$

Durbin-Watson = 0.1259

Nobs, Nvars = 34, 2

Variable Coefficient t-statistic t-probability

Const 3.473994 16.210840 0.000000

Zipf -2.030109 -11.037471 0.000000

Abrantes (Model2)
 Ordinary Least-squares Estimates
 R-squared = 0.6202
 Rbar-squared = 0.6044
 $\sigma^2 = 0.0524$
 Durbin-Watson = 0.4178
 Nobs, Nvars = 26, 2

 Variable Coefficient t-statistic t-probability
 Const 2.985786 9.428709 0.000000
 Zipf -1.470238 -6.260495 0.000002

Tomar (Model 2)
 Ordinary Least-squares Estimates
 R-squared = 0.7830
 Rbar-squared = 0.7764
 $\sigma^2 = 0.0314$
 Durbin-Watson = 0.1127
 Nobs, Nvars = 35, 2

 Variable Coefficient t-statistic t-probability
 Const 3.388989 16.294817 0.000000
 Zipf -1.706752 -10.911626 0.000000

Torres Novas (Model 2)
 Ordinary Least-squares Estimates
 R-squared = 0.7744
 Rbar-squared = 0.7673
 $\sigma^2 = 0.0325$
 Durbin-Watson = 0.1261
 Nobs, Nvars = 34, 2

 Variable Coefficient t-statistic t-probability
 Const 3.666301 15.035905 0.000000
 Zipf -2.145667 -10.480237 0.000000

Abrantes (Model2)
 Ordinary Least-squares Estimates
 R-squared = 0.6202
 Rbar-squared = 0.6044
 $\sigma^2 = 0.0524$
 Durbin-Watson = 0.4178
 Nobs, Nvars = 26, 2

 Variable Coefficient t-statistic t-probability
 Const 2.985786 9.428709 0.000000
 Zipf -1.470238 -6.260495 0.000002

Tomar (Model 2)
 Ordinary Least-squares Estimates
 R-squared = 0.7830
 Rbar-squared = 0.7764
 $\sigma^2 = 0.0314$
 Durbin-Watson = 0.1127
 Nobs, Nvars = 35, 2

 Variable Coefficient t-statistic t-probability
 Const 3.388989 16.294817 0.000000
 Zipf -1.706752 -10.911626 0.000000

Torres Novas (Model 2)
 Ordinary Least-squares Estimates
 R-squared = 0.7744
 Rbar-squared = 0.7673
 $\sigma^2 = 0.0325$
 Durbin-Watson = 0.1261
 Nobs, Nvars = 34, 2

 Variable Coefficient t-statistic t-probability
 Const 3.666301 15.035905 0.000000
 Zipf -2.145667 -10.480237 0.000000

Appendix 2 - Accessibility Surfaces

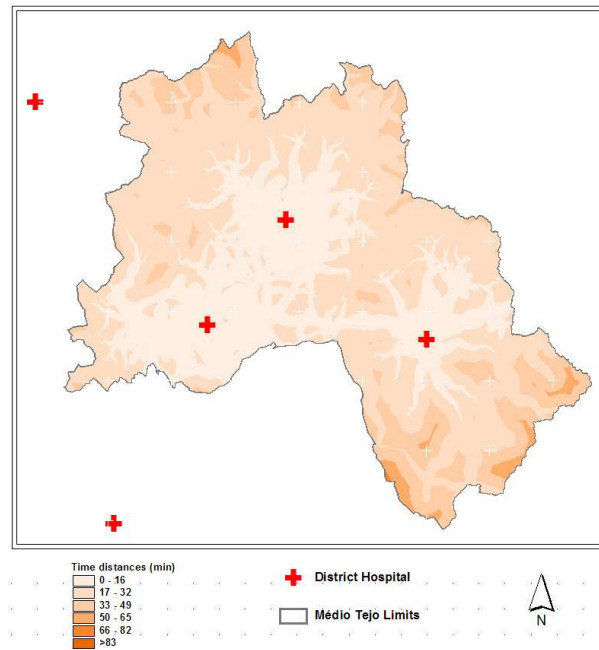


Figure 5: District Hospitals (Model 1)

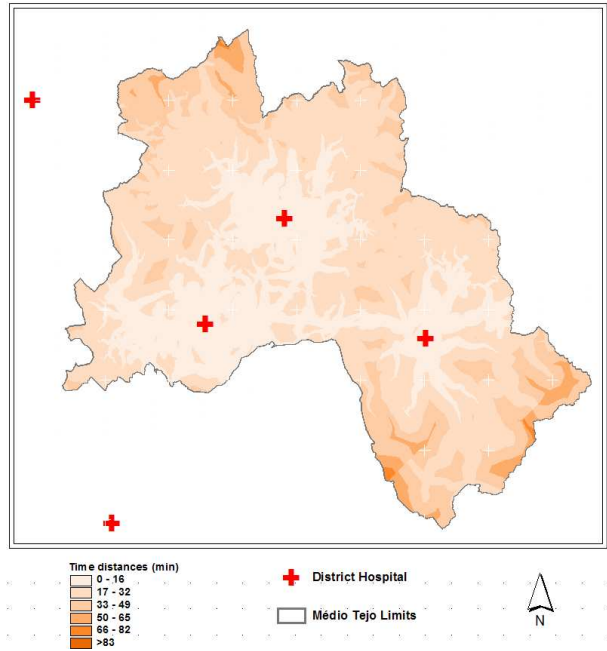


Figure 6: District Hospitals (Model 2)

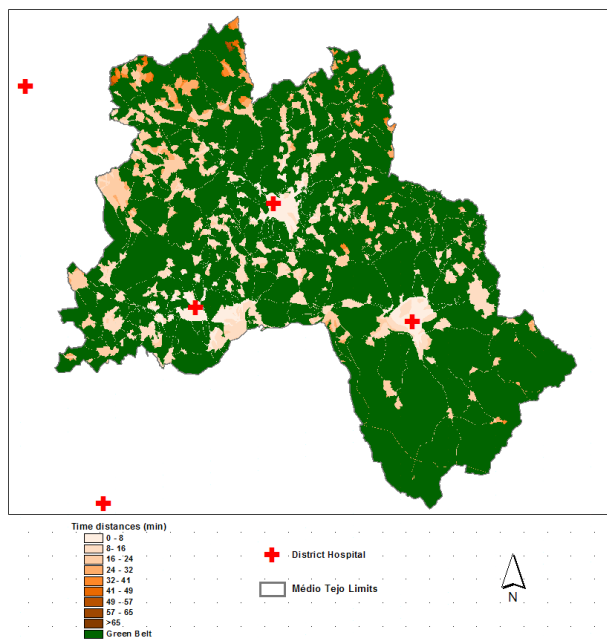


Figure 7: District Hospitals (Model 3)

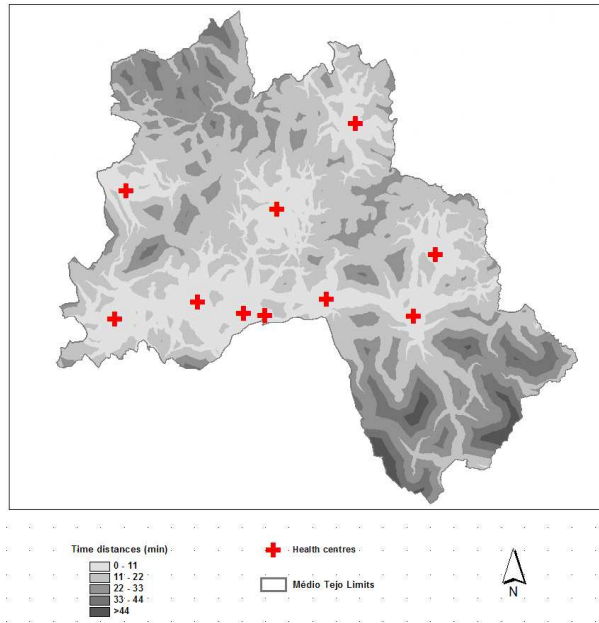


Figure 8: Health Centres (Model 1)

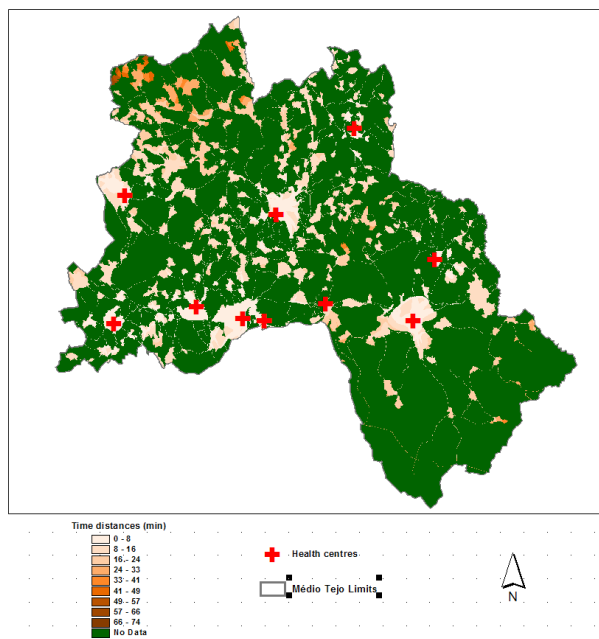


Figure 9: Health Centres (Model 2)

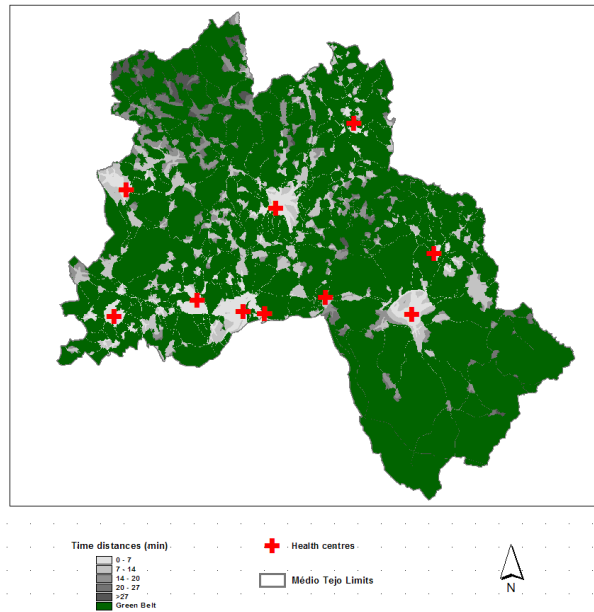


Figure 10: Health Centres (Model 3)

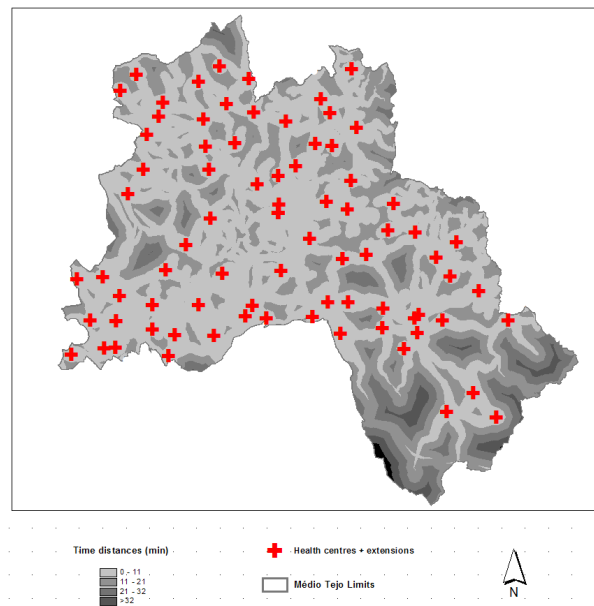


Figure 11: Health Centres + extensions (Model 1)

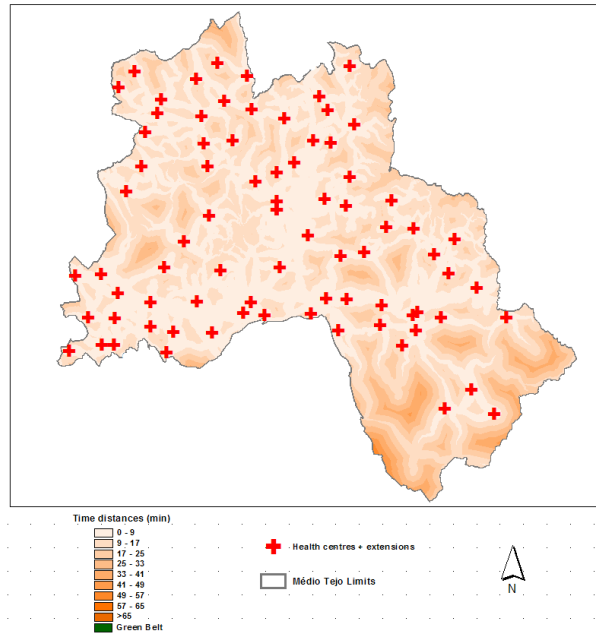


Figure 12: Health Centres + extensions (Model 2)

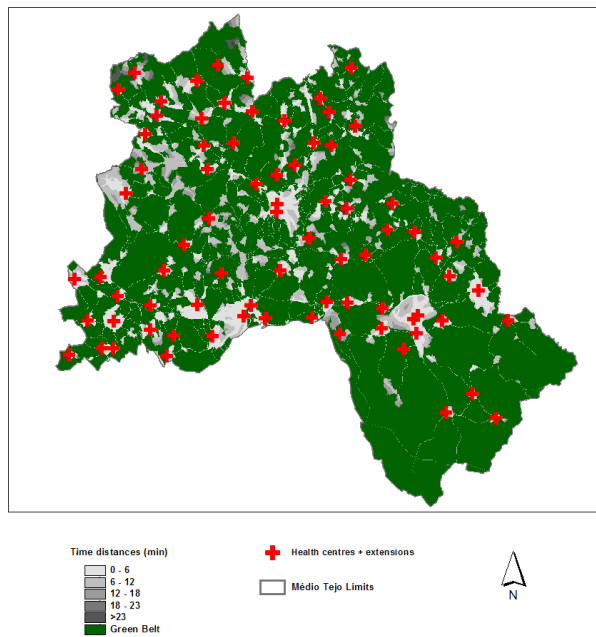


Figure 13: Health Centres + extensions (Model 3)